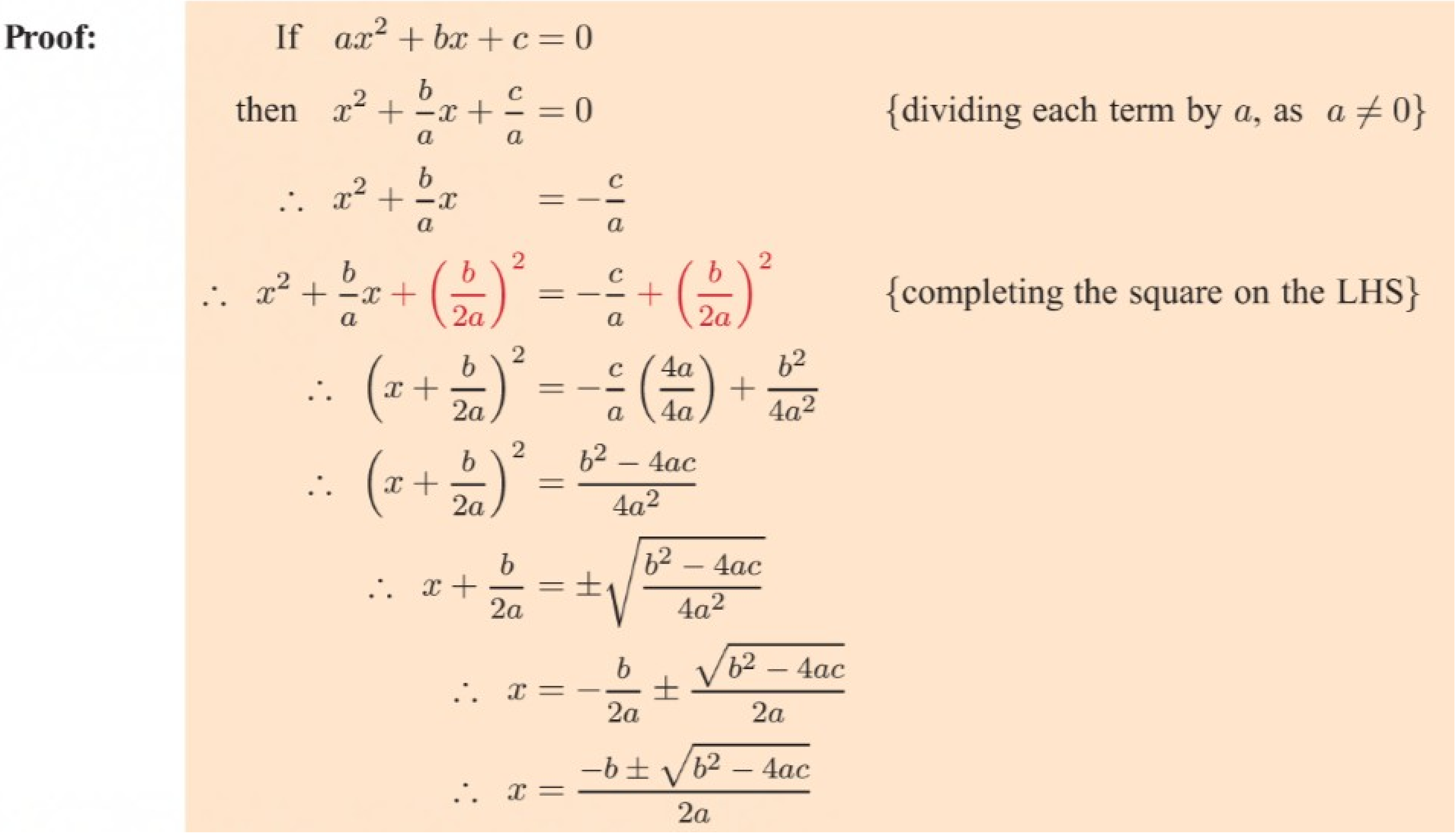
D THE QUADRATIC FORMULA

Many quadratic equations cannot be solved easily by factorisation, and completing the square is rather tedious. Consequently, the **quadratic formula** has been developed.

If $ax^2 + bx + c = 0$ where $a \neq 0$, then $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.



Proof:

$$
\begin{aligned}
\text{If}\quad ax^2+bx+c&=0\\
\text{then}\quad x^2+\frac{b}{a}x+\frac{c}{a}&=0 && \{\text{dividing each term by } a\text{, as } a\neq 0\}\\
\therefore\quad x^2+\frac{b}{a}x&=-\frac{c}{a}\\
\therefore\quad x^2+\frac{b}{a}x+\left(\frac{b}{2a}\right)^2&=-\frac{c}{a}+\left(\frac{b}{2a}\right)^2 && \{\text{completing the square on the LHS}\}\\
\therefore\quad \left(x+\frac{b}{2a}\right)^2&=-\frac{c}{a}\left(\frac{4a}{4a}\right)+\frac{b^2}{4a^2}\\
\therefore\quad \left(x+\frac{b}{2a}\right)^2&=\frac{b^2-4ac}{4a^2}\\
\therefore\quad x+\frac{b}{2a}&=\pm\sqrt{\frac{b^2-4ac}{4a^2}}\\
\therefore\quad x&=-\frac{b}{2a}\pm\frac{\sqrt{b^2-4ac}}{2a}\\
\therefore\quad x&=\frac{-b\pm\sqrt{b^2-4ac}}{2a}
\end{aligned}
$$

The quantity $b^2 - 4ac$ under the square root sign is called the **discriminant**.

The symbol **delta** Δ is used to represent the discriminant, so $\Delta = b^2 - 4ac$.

The quadratic formula becomes $x = \dfrac{-b \pm \sqrt{\Delta}}{2a}$ where Δ replaces $b^2 - 4ac$.

Δ must be $\geqslant 0$ for a quadratic equation to have real solutions.

If $\Delta > 0$, the quadratic equation has two distinct real solutions.

If $\Delta = 0$, the quadratic equation has one real solution, which is $x = -\dfrac{b}{2a}$.

Example 12

Self Tutor

Solve for x:

a $x^2 - 2x - 2 = 0$

b $2x^2 + 3x - 4 = 0$

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a $x^2 - 2x - 2 = 0$ has $a = 1,\ b = -2,\ c = -2$

$$\therefore\ x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-2)}}{2(1)}$$

$$\therefore\ x = \frac{2 \pm \sqrt{4 + 8}}{2}$$

$$\therefore\ x = \frac{2 \pm \sqrt{12}}{2}$$

$$\therefore\ x = \frac{2 \pm 2\sqrt{3}}{2}$$

$$\therefore\ x = 1 \pm \sqrt{3}$$

b $2x^2 + 3x - 4 = 0$ has $a = 2,\ b = 3,\ c = -4$

$$\therefore\ x = \frac{-3 \pm \sqrt{3^2 - 4(2)(-4)}}{2(2)}$$

$$\therefore\ x = \frac{-3 \pm \sqrt{9 + 32}}{4}$$

$$\therefore\ x = \frac{-3 \pm \sqrt{41}}{4}$$

1 Solve the following equations using: **i** factorisation **ii** the quadratic formula.

a $x^2 + 6x + 8 = 0$ **b** $x^2 - 10x + 25 = 0$ **c** $3x^2 - 7x - 6 = 0$

2 Use the quadratic formula to solve for x:

a $x^2 + x - 5 = 0$ **b** $x^2 - 5x + 5 = 0$ **c** $x^2 - 4x - 1 = 0$

d $3x^2 + 5x - 1 = 0$ **e** $-2x^2 + x + 7 = 0$ **f** $5x^2 - 8x + 1 = 0$

g $x^2 + 1 = 3x$ **h** $2x^2 = 2x + 3$ **i** $9x^2 = 6x + 1$

j $7x^2 = 5x + 1$ **k** $3x^2 + 2x = 2$ **l** $25x^2 + 1 = 20x$

3 Use the quadratic formula to solve for x:

a $(x+2)(x-1) = 5$ **b** $(x+1)^2 = 3 - x^2$ **c** $\dfrac{x-1}{x} = \dfrac{x}{3}$

d $x + \dfrac{1}{x+2} = 4$ **e** $3x - \dfrac{4}{x+1} = 10$ **f** $\dfrac{x+2}{x-1} = \dfrac{3x}{x+1}$

1 **a** **i, ii** $x = -2$ or -4 **b** **i, ii** $x = 5$

c **i, ii** $x = -\frac{2}{3}$ or 3

2 **a** $x = \dfrac{-1 \pm \sqrt{21}}{2}$ **b** $x = \dfrac{5 \pm \sqrt{5}}{2}$ **c** $x = 2 \pm \sqrt{5}$

d $x = \dfrac{-5 \pm \sqrt{37}}{6}$ **e** $x = \dfrac{1 \pm \sqrt{57}}{4}$ **f** $x = \dfrac{4 \pm \sqrt{11}}{5}$

g $x = \dfrac{3 \pm \sqrt{5}}{2}$ **h** $x = \dfrac{1 \pm \sqrt{7}}{2}$ **i** $x = \dfrac{1 \pm \sqrt{2}}{3}$

j $x = \dfrac{5 \pm \sqrt{53}}{14}$ **k** $x = \dfrac{-1 \pm \sqrt{7}}{3}$ **l** $x = \dfrac{2 \pm \sqrt{3}}{5}$

3 **a** $x = \dfrac{-1 \pm \sqrt{29}}{2}$ **b** $x = \dfrac{-1 \pm \sqrt{5}}{2}$ **c** no real solution

d $x = 1 \pm 2\sqrt{2}$ **e** $x = \dfrac{7 \pm \sqrt{217}}{6}$ **f** $x = \dfrac{3 \pm \sqrt{13}}{2}$